2018-2019 Guide

January 28 – March 29

Eureka

Module 5: *Fractions as Numbers on the Number Line*

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Module 5 Performance Overview

- Topic A opens Module 5 with students actively splitting different models of wholes into equal parts. They identify and count equal parts as *1 half, 1 fourth, 1 third, 1 sixth,* and *1 eighth* in unit form before an introduction to the unit fraction $1/b$.

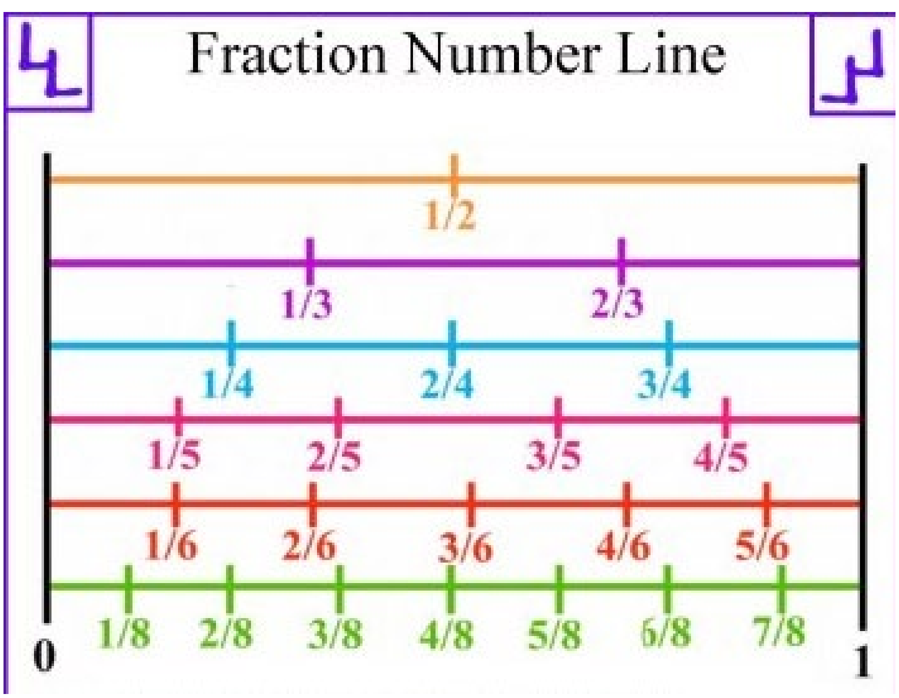
- In Topic B, students compare unit fractions and learn to build non-unit fractions with unit fractions as basic building blocks.

- In Topic C, students practice comparing unit fractions with fraction strips, specifying the whole and labeling fractions in relation to the number of equal parts in that whole.

- Students transfer their work to the number line in Topic D. They begin by using the interval from 0 to 1 as the whole. Continuing beyond the first interval, they partition, place, count, and compare fractions on the number line.

- In Topic E, they notice that some fractions with different units are placed at the exact same point on the number line, and therefore are equal. For example, 1/2, 2/4, 3/6, and 4/8 are equivalent fractions. Students recognize that whole numbers can be written as fractions.

- Topic F concludes the module with comparing fractions that have the same numerator. As they compare fractions by reasoning about their size, students understand that fractions with the same numerator and a larger denominator are actually smaller pieces of the whole.



Module 5: *Fractions as Numbers on the Number Line*

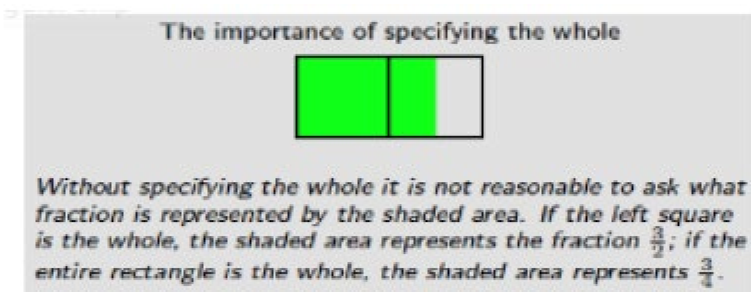
Pacing: January 28- March 29 **31 Days**		
Topic	**Lesson**	**Lesson Objective/ Supportive Videos**
Topic A: Partitioning A Whole into Equal Parts	Lesson 1	Specify and partition a whole into equal parts, identifying and counting unit fractions using concrete models. https://www.youtube.com/watch?v
	Lesson 2	Specify and partition a whole into equal parts, identifying and counting unit fractions by folding fraction strips. https://www.youtube.com/watch?v
	Lesson 3	Specify and partition a whole into equal parts, identifying and counting unit fractions by drawing pictorial area models. https://www.youtube.com/watch?v
Topic B: Unit Fractions and Their Relation to the Whole	Lesson 5	Partition a whole into equal parts and define the equal parts to identify the unit fraction numerically. https://www.youtube.com/watch?v
	Lesson 6	Build non-unit fractions less than one whole from unit fractions. https://www.youtube.com/watch?v
	Lesson 7	Identify and represent shaded and non-shaded parts of one whole as fractions. https://www.youtube.com/watch?v
	Lesson 8	Represent parts of one whole as fractions with number bonds. https://www.youtube.com/watch?v
	Lesson 9	Build and write fractions greater than one whole using unit fractions. https://www.youtube.com/watch?v
Topic C: Comparing Unit Fractions and Specifying the Whole	Lesson 10-11	Compare unit fractions by reasoning about their size using fraction strips. https://www.youtube.com/watch?v Compare unit fractions with different sized models representing the whole. https://www.youtube.com/watch?v
	Lesson 12	Specify the corresponding whole when presented with one equal part. https://www.youtube.com/watch?v
	Lesson 13	Identify a shaded fractional part in different ways depending on the designation of the whole. https://www.youtube.com/watch?v
Mid Module Assessment **February 5-7, 2019**		

Topic D: Fractions on the Number Line	Lesson 14	Place unit fractions on a number line with endpoints 0 and 1. https://www.youtube.com/watch?v
	Lesson 15	Place any fraction on a number line with endpoints 0 and 1. https://www.youtube.com/watch?v
	Lesson 16	Place whole number fractions and unit fractions between whole numbers on the number Line. https://www.youtube.com/watch?v
	Lesson 17	Practice placing various fractions on the number line. https://www.youtube.com/watch?v
	Lesson 18	Compare fractions and whole numbers on the number line by reasoning about their distance from 0. https://www.youtube.com/watch?v
Topic E: Equivalent Fractions	Lesson 20	Recognize and show that equivalent fractions have the same size, though not necessarily the same shape. https://www.youtube.com/watch?v
	Lesson 21	Recognize and show that equivalent fractions refer to the same point on the number line. https://www.youtube.com/watch?v
	Lesson 22	Generate simple equivalent fractions by using visual fraction models and the number line. https://www.youtube.com/watch?v
	Lesson 23	Generate simple equivalent fractions by using visual fraction models and the number line. https://www.youtube.com/watch?v
	Lesson 24	Express whole numbers as fractions and recognize equivalence with different units. https://www.youtube.com/watch?v
	Lesson 26	Decompose whole number fractions greater than 1 using whole number equivalence with various models. https://www.youtube.com/watch?v
	Lesson 27	Explain equivalence by manipulating units and reasoning about their size. https://www.youtube.com/watch?v
Topic F: Comparison, Order, and Size of Fractions	Lesson 28	Compare fractions with the same numerator pictorially. https://www.youtube.com/watch?v
	Lesson 29	Compare fractions with the same numerator using <, >, or = and use a model to reason about their size. https://www.youtube.com/watch?v
	Lesson 30	Partition various wholes precisely into equal parts using a number line method. https://www.youtube.com/watch?v
End Of Module Assessment March 28-29, 2019		

NJSLS Standards:

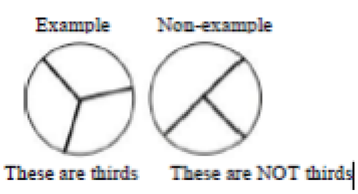
3.NF.1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

- This standard refers to the sharing of a whole being partitioned. Fraction models in third grade included only are (parts of a whole) models (circles, rectangles, squares) and number lines. Set models (parts of a groups) are not addressed in Third Grade.
- In 3.NF. 1 students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts and reasoning about one part of the whole, e.g. if a whole is partitioned into 4 equal parts then each part is $\frac{1}{4}$ of the whole, and 4 copies of that part make the whole.
- Students build fractions from unit fractions, seeing the numerator 3 of $\frac{3}{4}$ as saying that $\frac{3}{4}$ is the quantity you get by putting 3 pieces of $\frac{1}{4}$'s together. There is no need to introduce "im-proper fraction" initially.



<u>*Some important concepts related to developing understanding of fractions include:*</u>

- Understand fractional parts must be equal-sized

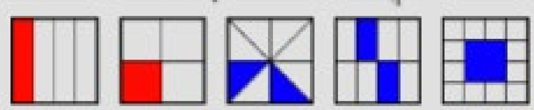


- The number of equal parts tells how many parts make a whole.
- As the number of equal parts in the whole increases, the size of the fractional part decreases.
- The size of the fractional part is relative to the whole. One-half of a small pizza is relatively smaller than one-half of a large pizza.
- When a whole is cut into equal parts, the denominator represents the number of equal parts.

- The numerator of a fraction is the count of the number of equal parts.
- Students can count one fourth, two fourths, three fourths.
- Students express fractions as fair sharing or, parts of a whole. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require them to create and reason about fair share.
- Initially, students can use an intuitive notion of “same size and same shape” (congruence) to explain why the parts are equal, e.g., when they divide a square into four equal squares or four equal rectangles. Students come to understand a more precise meaning for “equal parts” as “parts with equal measurements.”

Example:
When a ruler is partitioned into halves or quarters of an inch, students see that each subdivision has the same length.
In area models students reason about the area of a shaded region to decide what fraction of the whole it represents

Area representations of $\frac{1}{4}$

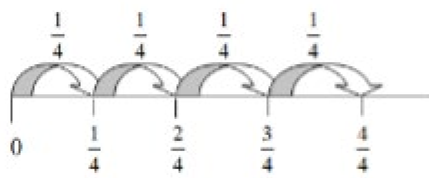


In each representation the square is the whole. The two squares on the left are divided into four parts that have the same size and shape, and so the same area. In the three squares on the right, the shaded area is $\frac{1}{4}$ of the whole area, even though it is not easily seen as one part in a division of the square into four parts of the same shape and size.

3.NF.2	Understand a fraction as a number on the number line, represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number.

- The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that $\frac{1}{2}$ is between 0 and 1). Students need ample experiences folding linear models (e.g., strings, sentence strips) to help them reason about and justify the location of fractions, such that $\frac{1}{2}$ lies exactly between 0 and 1.

- In the number line diagram, the space between 0 and 1 is divided (partitioned) into 4 equal parts. The distance from 0 to the first segments is 1 of the 4 parts from 0 to 1 or known as $\frac{1}{4}$. Similarly, the distance from 0 to the third segment is 3 parts that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction $\frac{3}{4}$



3.NF.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g. 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 = 1 at the same point of a number line diagram. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >,=, or <, and justify the conclusions e.g., by using visual fraction models.

- An important concept when comparing fractions is to *look at the size of the parts and the number of the parts.* For example, $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 same size whole is cut into 2 pieces.
- 3.NF.3a and 3.NF.3b: These standards call for students to use visual fraction models (area

models) and number lines to explore the idea of equivalent fractions. Students should only explore equivalent fractions using models, rather than using algorithms or procedures. This standard includes writing whole numbers as fractions. The concepts relates to fractions as division problems, where the fraction $\frac{3}{1}$ is 3 wholes divided into one group.

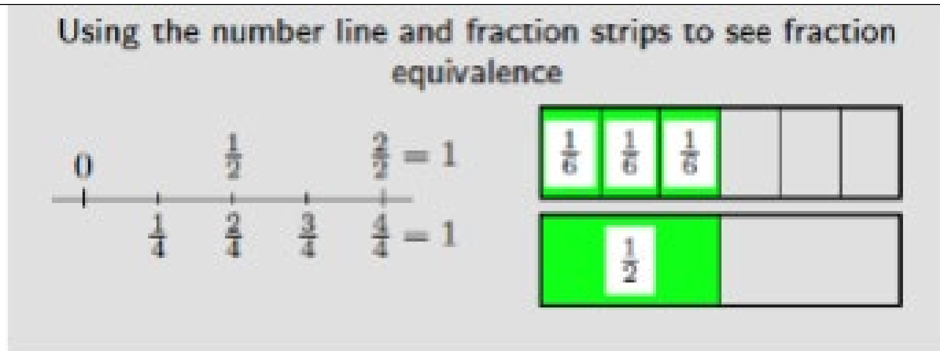
- **<u>This standard is the building block for later work where students divide set of objects into a specific number of groups. Students understand the meaning of a/1</u>**

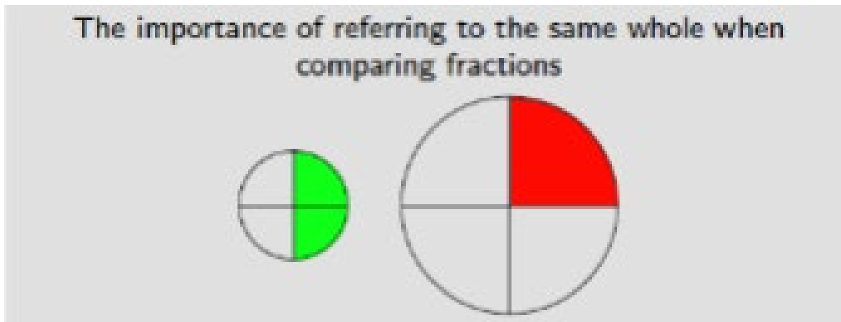
Example:
If 6 brownies are shared between 2 people, how many brownies would each person get?

- 3.NF.d: This standard involves comparing fractions with or without visual fraction models including number lines. Experiences should encourage students to reason about the size of pieces, such as $\frac{1}{3}$ of a cake being larger than $\frac{1}{4}$ of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths.
- In this standard, students should also reason that <u>*comparisons are only valid if the wholes are identical.*</u> For example, $\frac{1}{2}$ of a large pizza is a different amount than $\frac{1}{2}$ of a small pizza. Students should be given opportunities to discuss and reason about which $\frac{1}{2}$ is larger.
- Previously, in second grade, students compared lengths using a standard measure unit. In third grade, they build on this idea to compare fractions with the same denominator. They see that for fractions that have the same denominator, the underlying unit fractions are the same size, so the fraction with the greater numerator is greater because it is made of more unit fractions.

Example:
A segment from 0 to $\frac{3}{4}$ is shorter than the segment from 0 to $\frac{5}{4}$ because it measures 3 segments of $\frac{1}{4}$ as opposed to 5 segments of $\frac{1}{4}$. Therefore, $\frac{3}{4} < \frac{5}{4}$.

- Students also see that unit fractions with a larger denominator are smaller, by reasoning that in order for more (identical) pieces to make the same whole, the pieces must be smaller.
- From this idea, they reason that for fractions that have the same numerator, the fraction with the smaller denominator is greater. For example, $\frac{2}{5} > \frac{2}{7}$, because $\frac{1}{7} < \frac{1}{5}$, so 2 pieces of $\frac{1}{7}$ is less than 2 pieces of $\frac{1}{5}$. As with equivalence of fractions, it is important to make sure rgar each fraction refers to the same whole when comparing fractions.



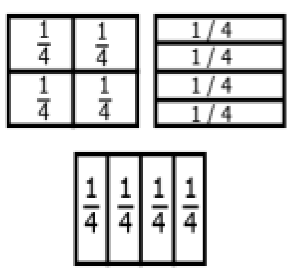


A student might think that $\frac{1}{4} > \frac{1}{2}$, because a fourth of the pizza on the right is bigger than a half of the pizza on the left.

3.G.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

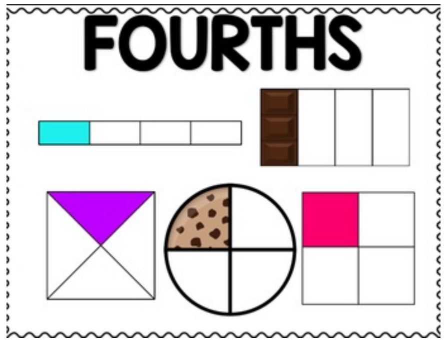
- Teacher gives a variety of shapes and students partition them into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.

Example:



- Help students learn to describe how a shape can be partitioned into four parts with equal area. Talk about the area of each part as $\frac{1}{4}$ of the area of the shape.
- Students may be confused with the concept that equal shares of identical wholes may not have the same shape. Provide additional experiences about equal shares with different

shapes help them understand this confusing concept.



Common multiplication and division situations. [1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	**3 x ? = 18**, and **18 ÷ 3 = ?**	**? x 6 = 18**, and **18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS [2], AREA [3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	**a x b = ?**	**a x ? = p** and **p ÷ a = ?**	**? x b = p**, and **p ÷ b = ?**

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Module 5 Assessment / Authentic Assessment Recommended Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Eureka Math Module 5: *Fractions as Numbers on the Number Line*			
Authentic Assessment: Identifying a Fraction	3.NF.1	30 mins	Individual
Authentic Assessment: Equivalent Fractions	3.NF.3	30 mins	Individual
Optional Mid-Module Assessment	3.NF.1 3.NF.3 3.G.2	1 Block	Individual
Optional End of Module Assessment	3.NF.1 3.NF.2 3.NF.3 3.G.2	1 Block	Individual

Third Grade Ideal Math Block

Getting Ready: Whole Group
Number Talk/ Number Strings
7-10 min.

Fluency: Whole Group
Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

Concept Development: Individual/partner/whole
Instruction & Strategic Problem Set Questions

Student Debrief: Independent
Exit Ticket

50-60 min.

CENTERS/STATIONS:
Pairs / Small Group/ Individual
DIFFERENTIATED activities designed to **RETEACH, REMEDIATE, ENRICH** student's understanding of concepts.
15-20 min.

M: Meet with the teacher	A: Application/ Problem Solving	T: Technology	H: Hands on Activities

SUMMARY: Whole Group
Lesson Closure: Student Reflection; Real Life Connections to Concept
Up to 5 min.

Eureka Lesson Structure:
Fluency: • Sprints • Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models • Whiteboard Exchange
Application Problem: • Engage students in using the RDW Process • Sequence problems from simple to complex and adjust based on students' responses • Facilitate share and critique of various explanations, representations, and/or examples.
Concept Development: (largest chunk of time) Instruction: • Maintain overall alignment with the objectives and suggested pacing and structure. • Use of tools, precise mathematical language, and/or models • Balance teacher talk with opportunities for peer share and/or collaboration • Generate next steps by watching and listening for understanding Problem Set: (Individual, partner, or group) • Allow for independent practice and productive struggle • Assign problems strategically to differentiate practice as needed • Create and assign remedial sequences as needed
Student Debrief: • licit students thinking, prompt reflection, and promote metacognition through student centered discussion • Culminate with students' verbal articulation of their learning for the day • Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

PARCC Assessment Evidence/Clarification Statements			
CCSS	**Evidence Statement**	**Clarification**	**MP**
3.NF.1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.	• Tasks do not involve the number line. • Fractions equivalent to whole numbers are limited to 0 through 5. • Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8	MP 2
3.NF.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.	• Fractions may be greater than 1. • Fractions equivalent to whole numbers are limited to 0 through 5. • Fractions equal whole numbers in 20% of these tasks. • Tasks have "thin context"2 or no context. • Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8.	MP 5
3.NF.3a -1	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size	• Tasks do not involve the number line. • Fractions equivalent to whole numbers are limited to 0 through 5. • Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. • The explanation aspect of 3.NF.3 is not assessed here.	MP 5

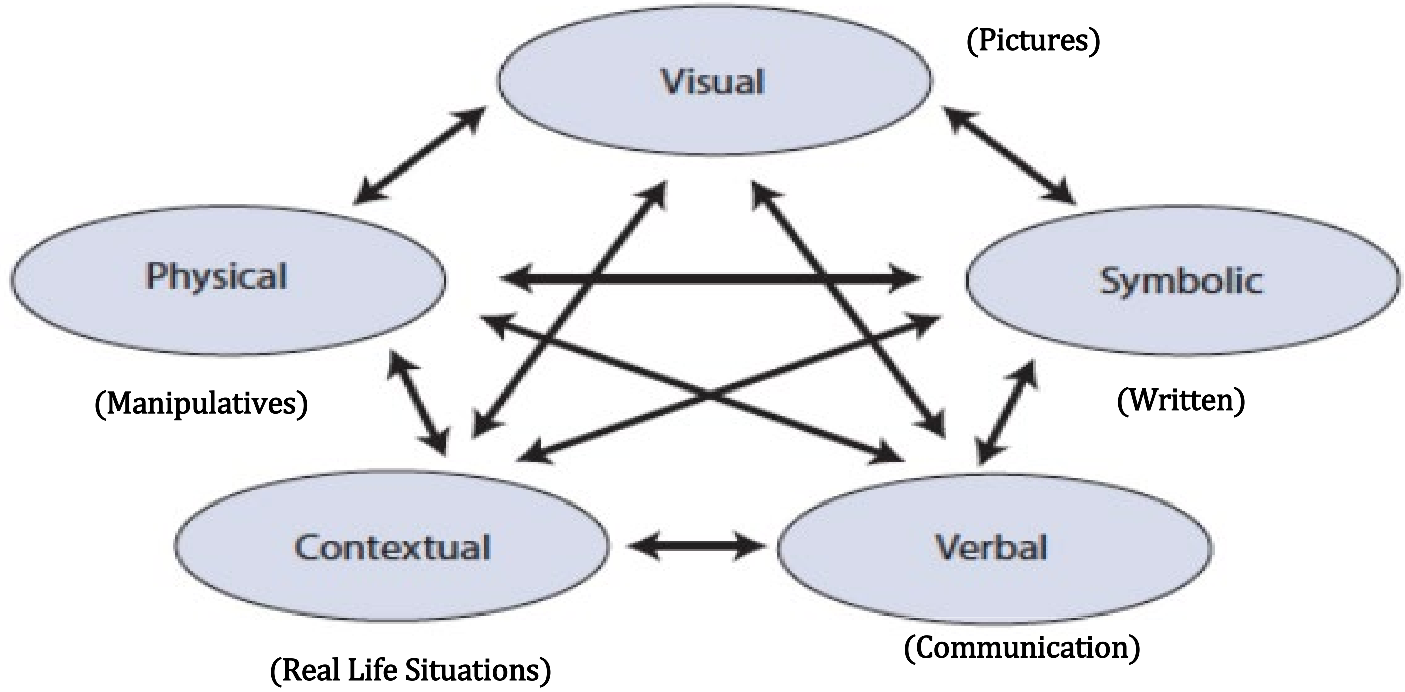
3.NF.3a -2	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same point on a number line	• Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. • Fractions equivalent to whole numbers are limited to 0 through 5. • The explanation aspect of 3.NF.3 is not assessed here.	MP 5
3.NF.3b -1	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3).	• Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. • Fractions equivalent to whole numbers are limited to 0 through 5. • The explanation aspect of 3.NF.3 is not assessed here.	MP 7
3.NF.3c	Explain equivalence of fractions in special cases and compare fractions by reasoning about their size. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.	• Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. • Fractions equivalent to whole numbers are limited to 0 through 5. • The explanation aspect of 3.NF.3 is not assessed here.	MP 3, 5, 7
3.NF.3d	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or	Tasks are limited to fractions with denominators 2, 3, 4, 6, and 8. ii) Fractions equivalent to whole numbers are limited to 0 through 5. iii) Justifying is not assessed here. For this aspect of 3.NF.3d, see 3.C.3-1 and 3.C.4-4. iv) Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.	MP 7

3.NF.A.Int.1	In a contextual situation involving a whole number and two fractions not equal to a whole number, represent all three numbers on a number line diagram, then choose the fraction closest in value to the whole number.	• Fractions equivalent to whole numbers are limited to 0 through 5. • Fraction denominators are limited to 2, 3, 4, 6 and 8.	MP 2,4,5

Student Name: ______________________________ Task: _____________________ School: ________ Teacher: __________ Date: ________

"I CAN….."	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a class-mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my think-ing.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: "Doing Stage": Physical manipulation of objects to solve math problems.

Pictorial: "Seeing Stage": Use of imaged to represent objects when solving math problems.

Abstract: "Symbolic Stage": Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.
DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?
WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

100 questions that promote Mathematical Discourse

Help students **work together** to make sense of mathematics

1. What **strategy** did you use?
2. Do you **agree**?
3. Do you **disagree**?
4. Would you **ask the rest of the class** that question?
5. Could you **share your method** with the class?
6. What part of what he said **do you understand**?
7. Would someone like to **share** ___?
8. Can you **convince the rest of us** that your answer makes sense?
9. **What do others think** about what [student] said?
10. Can someone **retell or restate** [student]'s explanation?
11. Did you **work together**? In what way?
12. Would anyone like to **add to what was said**?
13. Have you **discussed** this with your group? With others?
14. Did anyone get a **different answer**?
15. **Where** would you go for **help**?
16. **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
17. How could you help another student **without telling them the answer**?
18. **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

19. Is this a **reasonable answer**?
20. Does that make **sense**?
21. **Why** do you think that? Why is that true?
22. Can you **draw a picture or make a model** to show that?
23. **How** did you reach that conclusion?
24. Does anyone want to **revise** his or her answer?
25. **How were you sure** your answer was right?

Ready

Help students learn to **reason mathematically**

26. **How did you begin** to think about this problem?
27. What is **another way** you could solve this problem?
28. How could you **prove** ______?
29. Can you **explain how your answer is different from or the same as** [student]'s answer?
30. Let's **break the problem into parts.** What would the parts be?
31. Can you **explain this part more specifically**?
32. Does that **always work**?
33. Can you think of a case where that **wouldn't work**?
34. How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35. What do you need to do **next**?
36. What have you **accomplished**?
37. What are your **strengths and weaknesses**?
38. Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39. What is this problem about? What can you **tell me about it**?
40. Do you need to **define or set limits** for the problem?
41. How would you **interpret** that?
42. Could you **reword that in simpler terms**?
43. Is there something that can be **eliminated** or that is **missing**?
44. Could you **explain** what the problem is asking?
45. What **assumptions** do you have to make?
46. What do you **know** about this part?
47. Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74 What is the **relationship** between ___ and ___?

75 Have we ever solved a problem **like this before**?

76 What uses of mathematics did you find in the **newspaper** last night?

77 What is the **same**?

78 What is **different**?

79 Did you use skills or build on concepts that were **not necessarily mathematical**?

80 Which **skills or concepts** did you use?

81 What **ideas** have we explored before that were useful in solving this problem?

82 Is there a **pattern**?

83 **Where else** would this strategy be useful?

84 How does this **relate** to ___?

85 Is there a **general rule**?

86 Is there a **real-life situation** where this could be used?

87 How would your method work with **other problems**?

88 What other problem does this seem to **lead to**?

Help students **persevere**

89 Have you tried making a **guess**?

90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

93 Give me another **related problem**. Is there an easier problem?

94 How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95 What was **one thing you learned** (or two, or more)?

96 Did you **notice any patterns**? If so, describe them.

97 What **mathematics topics** were used in this investigation?

98 What were the **mathematical ideas** in this problem?

99 What is mathematically **different about these two situations**?

100 What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation

3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)
3.NBT.A.2: Add/subtract within 1000
4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm
5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

Mathematical Proficiency

To be mathematically proficient, a student must have:

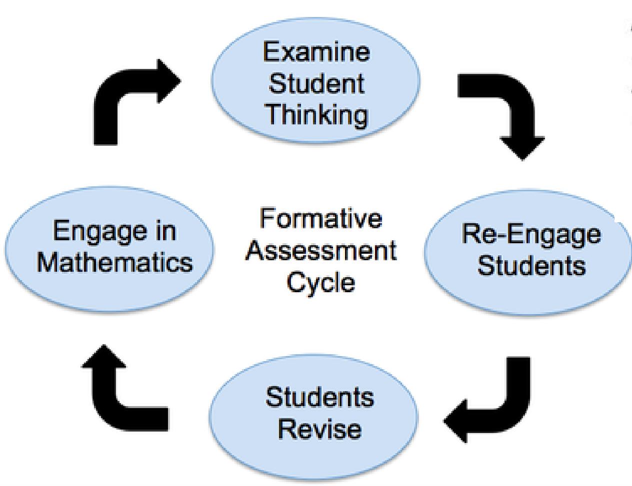
- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)



Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The Standards for Mathematical Practice:

Describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1	**Make sense of problems and persevere in solving them** In **third** grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try approaches. They often will use another method to check their answers.
2	**Reason abstractly and quantitatively** In **third** grade, students should recognize that number represents a specific quantity. They connect quantity to written symbols and create logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities
3	**Construct viable arguments and critique the reasoning of others** In **third** grade, mathematically proficient students may construct viable arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, "How did you get that?" and "Why is it true?" They explain their thinking to others and respond to others' thinking.
4	**Model with mathematics** Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language) drawing pictures, using objects, acting out, making chart, list, or graph, creating equations etc…Students need opportunities to connect different representations and explain the connections. They should be able to use all of the representations as needed. **Third** graders should evaluate their results in the context of the situation and reflect whether the results make any sense.
5	**Use appropriate tools strategically** **Third** graders should consider all the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For example, they might use graph paper to find all possible rectangles with the given perimeter. They compile all possibilities

	into an organized list or a table, and determine whether they all have the possible rectangles.
6	**Attend to precision** Mathematical proficient **third** graders develop their mathematical communication skills; they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying their units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle the record their answer in square units.
7	**Look for and make use of structure** In **third** grade, students should look closely to discover a pattern of structure. For example, students properties of operations as strategies to multiply and divide. (commutative and distributive properties.
8	**Look for and express regularity in repeated reasoning** Mathematically proficient students in third **grade** should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7x8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.

In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.
In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: ____ Time: ___________

NJSLS.:

____

____

Objective(s): By the end of this task, I will be able to:

- ____
- ____
- ____

Task(s):

- ____
- ____
- ____
- ____

Exit Ticket:

- ____
- ____
- ____

MATH WORKSTATION SCHEDULE

Week of: ________________

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OBSERVATIONAL DATA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

Third Grade PLD Rubric

Got It Evidence shows that the student essentially has the target concept or big math idea.		Not There Yet Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% Distinguished command	**PLD Level 4: 89% Strong Command**	**PLD Level 3: 79% Moderate Command**	**PLD Level 2: 69% Partial Command**	**PLD Level 1: 59% Little Command**
Student work shows **distinguished levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction relationship • Use of math vocabulary Response includes an **efficient and logical progression** of mathematical reasoning and understanding.	Student work shows **strong levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction relationship • Use of math vocabulary Response includes a **logical progression** of mathematical reasoning and understanding.	Student work shows **moderate levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction relationship • Use of math vocabulary Response includes a **logical but incomplete progression** of mathematical reasoning and understanding. Contains **minor errors.**	Student work shows **partial understanding** of the mathematics. Student **constructs** and **communicates** an **incomplete response** based on student's attempts of explanations/ reasoning using the: • properties of operations • relationship between addition and subtraction relationship • Use of math vocabulary Response includes an **incomplete or illogical progression** of mathematical reasoning and understanding.	Student work shows **little understanding** of the mathematics. Student **attempts** to **constructs** and **communicates** a response using the: • properties of operations • relationship between addition and subtraction relationship • Use of math vocabulary Response includes **limited evidence of the progression** of mathematical reasoning and understanding.
5 points	**4 points**	**3 points**	**2 points**	**1 point**

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

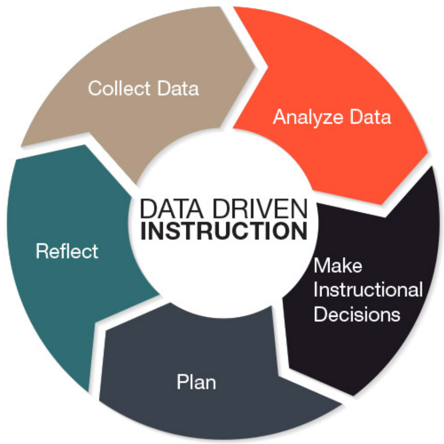
Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



Data Analysis Form **School:** ________________ **Teacher:** ________________________ **Date:** _____________

Assessment: ____ **NJSLS:** ____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

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MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews[1].
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended.*
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)[2].
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

GRADES K-2

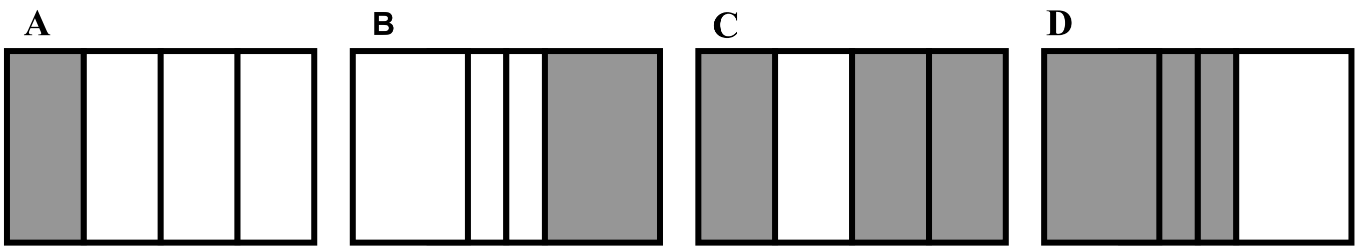
Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students <u>should retain ALL of their current artifacts</u> in their Mathematics Portfoli

Authentic Assessment– Identifying a Fration

Name: Date: 3.NF.1

Kai shaded four pieces of paper with a gray crayon.



Which piece of paper is 3/4 shaded?

____

Use what you know about fractions to explain why your answer is correct.

Authentic Assessment Scoring Rubric – Identifying a Fraction

3.NF.1: Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

Teacher notes:
Student learning targets for this task may include:

- I can explain any unit fraction as one part of a whole.
- I can explain any fraction (a/b) as "a" (numerator) being the numbers of parts and "b" (denominator) as the total number of equal parts in the whole.
- I can represent a fraction and explain my representation.

One meaning for a fraction is a number that represents a part of the whole. When a fraction is used to describe part of a region, the whole needs to be divided into equal parts. When dividing a region into equal parts, it is not necessary that the parts have the same shape as long as they have the same area.

Students who demonstrate mastery can identify that C shows ¾ shaded. They should also be able to explain that they counted the total number of equal parts to find the denominator and then counted the number of pieces that had been shaded to find the numerator.

Students who demonstrate partial mastery may choose the correct picture, but may not be able to explain how they found the answer.

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on explanations/reasoning using the: • Understanding of equal parts • Understanding of numerator and denominator Response includes an **<u>efficient</u>** and logical progression of steps.	Clearly constructs and communicates a complete response based on explanations/ reasoning using the: • Understanding of equal parts • Understanding of numerator and denominator Response includes a **<u>logical</u>** progression of steps	Constructs and communicates a complete response based on explanations/reasoning using the: • Understanding of equal parts • Understanding of numerator and denominator Response includes a **<u>logical but incomplete</u>** progression of steps. Minor calculation errors.	Constructs and communicates an incomplete response based on explanations/reasoning using the: • Understanding of equal parts • Understanding of numerator and denominator Response includes an **<u>incomplete or Illogical</u>** progression of steps.	The student shows no work or justification

Authentic Assessment– Equivalent Fractions

Mrs. Caha asked her class to write fractions on their whiteboards that were equivalent to $\frac{1}{2}$.

Tell if each student's fraction is equivalent to Mrs. Caha's fraction and show how you know.

Gloria : $\frac{3}{4}$	CIRCLE ONE ○ Yes ○ No	Show how you know:
Isaiah: $\frac{2}{3}$	CIRCLE ONE ○ Yes ○ No	Show how you know:
Thomas: $\frac{4}{8}$	CIRCLE ONE ○ Yes ○ No	Show how you know:

Authentic Assessment Scoring Rubric – Equivalent Fractions

<u>**3.NF.A.3**</u>: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on explanations/reasoning using the: • Number line, visual model, reasoning about size Response includes an <u>**efficient**</u> and logical progression of steps.	Clearly constructs and communicates a complete response based on explanations/ reasoning using the: • Number line, visual model, reasoning about size Response includes a <u>**logical**</u> progression of steps	Constructs and communicates a complete response based on explanations/ reasoning using the: • Number line, visual model, reasoning about size Response includes a <u>**logical but incomplete**</u> progression of steps. Minor calculation errors.	Constructs and communicates an incomplete response based on explanations/ reasoning using the: • Number line, visual model, reasoning about size Response includes an <u>**incomplete or Illogical**</u> progression of steps.	The student shows no work or justification

Resources

Engage NY
http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

Common Core Tools
http://commoncoretools.me/
http://www.ccsstoolbox.com/
http://www.achievethecore.org/steal-these-tools

Achieve the Core
http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000

http://www.thinkingblocks.com/

Illustrative Math Project :http://illustrativemathematics.org/standards/k8

Inside Mathematics: http://www.insidemathematics.org/index.php/tools-for-teachers

Sample Balance Math Tasks: http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/

Georgia Department of Education:https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx

Gates Foundations Tasks:http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf

Minnesota STEM Teachers' Center:
http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships

Singapore Math Tests K-12: http://www.misskoh.com

Mobymax.com: http://www.mobymax.com

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

"Eureka Math" *Great Minds.* 2018 < https://greatminds.org/account/products>